Denoising Accuracy of Adaptive ICI-Based Estimators With Regards to Sampling Rate

Jonatan Lerga, Nicoletta Saulig, Martina Žuškin, Ante Panjkota

***Abstract*—This paper presents study on denoising accuracy of adaptive temporal filtering methods based on the intersection of confidence intervals (ICI) rule and relative intersection of confidence intervals (RICI) rule with regards to signal sampling rate. The original ICI-based and the improved RICI-based method were tested on four signal classes for a range of signal to noise ratios (SNRs). Denoising accuracy, with respect to signal sampling rate, was measured in terms of the reductions in root mean squared error (RMSE) and mean absolute error (MAE). Extensive simulations showed that the data-driven RICI method outperformed the original ICI method reducing the RSME by up 79.6% and the MAE by up to 86.1%. It is important to note that both methods, especially the RICI method, exhibit significant estimation accuracy improvement in case of signals with higher sampling rates.**

***Index Terms*—Intersection of confidence intervals (ICI) rule, relative intersection of confidence intervals (ICI) rule, signal denoising, sampling rate**

I. Introduction

Noise is inevitable in real-world data and may be negligible only in cases of high signal to noise ratios (SNRs). However, in most practical scenarios, noise corrupts analyzed signals in a significant manner. Hence, its removal is a crucial step proceeding any further signal processing and analysis. Thus, denoising preprocessing is found in a wide range of applications, such as data mining, video coding, (medical) image analysis, seismology, radar, sonar, speech processing, astronomy and many more.

In particular, additive white Gaussian noise (AWGN), due to its nature, adds high frequency components to considered signal. Hence, one of denoising approaches is to remove the AWGN in transform domain (using the Fourier or wavelet transforms). However, in some practical applications utilizing low-pass filters in Fourier domain cause also the removal of fine details which may carry key information. On the other hand, temporal/spatial filtering methods perform noise suppression on raw signal in original signal domain and hence temporal filtering does not require loss-less inverse transform procedure.

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Note that temporal filters may be divided to fixed and adaptive. Fixed size filters, in order to efficiently reduce noise, require a prior knowledge on both signal and noise [1]. On the other hand, adaptive filters are able to reconfigure themselves by tracking non-stationary changes in signal and to adjust to input statistics (meaning that little or no prior knowledge on noise or signal is required) [1].

This paper presents an adaptive temporal filtering method based on the intersection of confidence intervals (ICI) rule and its improved version (called relative intersection of confidence intervals (RICI) rule) applied to signal denoising. The original ICI method was first proposed in [2], and later was applied to adaptive bandwidths selection and denoising in [3], [4], [5]. The original ICI method was shown to efficiently adapt to unknown smoothness of the signal ensuring close to optimal bias-to-variance tradeoff [6]. One of the advantages of the method is requiring only the estimation of noise variance, and not the bias (nor higher derivatives of the estimate needed for bias estimation, often required by numerous plug-in methods [3]).

On the other hand, the main shortcoming of the ICI method is its sensitivity to suboptimal threshold Γ selection. Namely, small Γ values result in poor denoising, while, on the other hand, large Γ values oversmooth denoised signal. This problem was solved by the RICI method which introduced additional criterion for optimal window width selection that allowed choosing arbitrary large Γ values with oversmoothing being avoided by additional threshold.

The paper provides study on the denoising quality of both methods with regards to signal sampling rate. For indices of denoising performances the reduction in root mean squared error (RMSE) and mean absolute error (MAE) were used.

The rest of the paper is organized as follows. Section II presents theoretical background of the ICI and the improved RICI method. Their denoising performances, in terms of estimation error reduction, with respect to signal sampling frequency are elaborated in Section III. Conclusion is found in Section IV.

II. Original and improved ICI estimator

A. ICI-based estimator

Let us model discrete noisy signal $z(t_i)$ corrupted by the AWGN as:

$$z(t_i) = y(t_i) + \varepsilon(t_i), \tag{1}$$

where $y(t_i)$ is the original discrete signal, $\varepsilon(t_i)$ is zero-mean normally distributed error ($E\{\varepsilon\} = 0$ and $E\{\varepsilon^2\} = \sigma^2$), and

Δ is sampling period such that time $t_i = i\cdot\Delta$, $i = 1, 2, \cdots, N$ (sampling frequency is $f_s = \frac{1}{\Delta}$).

The goal of the denoising procedure is to obtain discrete estimate $\hat{y}(t_i)$, as close as possible to the noise-free signal $y(t_i)$, with estimation error:

$$e(t_i, h) = y(t_i) - \hat{y}(t_i, h), \quad (2)$$

being minimal. According to [7], $e(t_i, h)$ can be expressed as:

$$|e(t_i, h)| \quad \leq \quad b(t_i, h) + |\zeta(t_i, h)|, \quad (3)$$

where $b(t_i, h)$ is estimation bias, h is estimation window size, and $\zeta(t_i, h)$ is normally distributed random error ($N(0, s(t_i, h))$). The following inequality, as shown in [3], holds:

$$|\zeta(t_i, h)| \quad \leq \quad \chi_{1-\beta/2} \cdot s(t_i, h), \quad (4)$$

with probability $p = 1 - \beta$ (where $\chi_{1-\beta/2}$ stands for the $(1 - \beta/2)$th quantile of the standard Gaussian distribution). By introducing Eq. (4) to Eq. (3) the following inequity is obtained:

$$|e(t_i, h)| \quad \leq \quad b(t_i, h) + \chi_{1-\beta/2} \cdot s(t_i, h). \quad (5)$$

In order to achieve desirable denoising performance we are to find optimal window size h_0 which ensures best compromise (defined by α) between bias $b(t_i, h)$ and random error [8]:

$$h_0 \quad = \quad \max\left\{h : \frac{b(t_i, h)}{\chi_{1-\beta/2} \cdot s(t_i, h)} \leq \alpha\right\}. \quad (6)$$

It was shown in [8], for $h \leq h_0$:

$$b(t_i, h) \quad \leq \quad \alpha \cdot \chi_{1-\beta/2} \cdot s(t_i, h), \quad (7)$$

which lead to [3]:

$$|e(t_i, h)| \quad \leq \quad (1 + \alpha) \cdot \chi_{1-\beta/2} \cdot s(t_i, h), \quad (8)$$

or written in another form:

$$|y(t_i) - \hat{y}(t_i, h)| \quad \leq \quad \Gamma \cdot s(t_i, h), \quad (9)$$

where $\Gamma = (1 + \alpha) \cdot \chi_{1-\beta/2}$ is preset threshold. Thus, Eq. (9) can be expressed as:

$$\hat{y}(t_i, h) - \Gamma \cdot s(t_i, h) \leq y(t_i) \leq \hat{y}(t_i, h) + \Gamma \cdot s(t_i, h). \quad (10)$$

Hence, upper and lower boundaries of confidence interval $D(t_i, h)$ limits can be expressed as:

$$U(t_i, h) = \hat{y}(t_i, h) + \Gamma \cdot s(t_i, h), \quad (11)$$

and

$$L(t_i, h) = \hat{y}(t_i, h) - \Gamma \cdot s(t_i, h). \quad (12)$$

Here we need an algorithm to test the hypothesis $h \leq h_0$ for a range of window widths $H = \{h_1 < h_2 < \cdots < h_L\}$ resulting in the h_{j^+} as close to the optimal h_0 as possible. The motivation for the algorithm, called the ICI rule, cames from Eq. (9) suggesting that $y(t_i) \in D(t_i, h)$ for $h_i \leq h_0$. Or, in other words, intersection of $D(t_i, h_i)$, $1 \leq i \leq j$ is nonempty and has at least one common point (that point is $y(t_i)$) for $h_i \leq h_0$. Thus, h_{j^+} can be found by increasing h_i as long as the intersection of $D(t_i, h_i)$, $1 \leq i \leq j$, is nonempty. Hence, the largest j (denoted as j^+) for which exists nonempty intersection of all previous confidence intervals $D(t_i, h_i)$, $1 \leq i \leq j^+$ defines the proper h_{j^+} used for calculation of the corresponding estimate $\hat{y}(t_i, h_{j^+})$.

Thus, the intersection of $D(t_i, h_i)$, $1 \leq i \leq j$, can be defined as:

$$\overline{L}(t_i, h_j) \quad \leq \quad \underline{U}(t_i, h_j), \quad (13)$$

where $\overline{L}(t_i, h_j)$ is the largest lower boundary, defined as:

$$\overline{L}(t_i, h_j) = \max\{\overline{L}(t_i, h_{j-1}), L(t_i, h_j)\}, \quad j = 1, 2, \cdots, L, \quad (14)$$

and $\underline{U}(t_i, h_j)$ is the smallest upper boundary, defined as:

$$\underline{U}(t_i, h_j) = \min\{\underline{U}(t_i, h_{j-1}), U(t_i, h_j)\}, \quad j = 1, 2, \cdots, L. \quad (15)$$

Note that performances of the ICI-based denoising are highly dependant on the threshold Γ. Namely, large Γ causes signal oversmoothing ($h_{j^+} > h_0$). On the other hand, small Γ leads to signal undersmoothing ($h_{j^+} < h_0$) [9]. Furthermore, as shown in [3], it is difficult to find the optimal, data-dependant Γ from a theoretical analysis. In order to solve this problem, the following section introduces a modification of the ICI method (called RICI rule) robust to suboptimal threshold Γ selection. In addition, the RICI method was shown to significantly outperform the original ICI in terms of denoising quality.

B. RICI-based estimator

The motivation behind the improved ICI method is to avoid signal oversmoothing (caused by too large Γ values) by introducing an additional criterion besides tracking the non-emptiness of confidence intervals intersection. Namely, the RICI method also takes into account the amount of this overlapping, defined as:

$$O(t_i, h) \quad = \quad \underline{U}(t_i, h) - \overline{L}(t_i, h). \quad (16)$$

Let us here define $R(t_i, h)$ as a ratio of the overlapping and size of the considered confidence interval $D(t_i, h)$:

$$R(t_i, h) \quad = \quad \frac{O(t_i, h)}{U(t_i, h) - L(t_i, h)}, \quad (17)$$

$$\quad = \quad \frac{\underline{U}(t_i, h) - \overline{L}(t_i, h)}{U(t_i, h) - L(t_i, h)}. \quad (18)$$

Note that minimal $R(t_i, h) = 0$ in case of the intersection of confidence intervals being empty. On the other hand, maximal value of $R(t_i, h) = 1$ is obtained when intersection of all previous confidence intervals is inside the considered confidence interval. Being that $R(t_i, h) \in [0, 1]$, the RICI rule introduces additional threshold R_c, such that:

$$R(t_i, h) \quad \geq \quad R_c, \quad (19)$$

where $R_c \in [0, 1]$. Note that the ICI method is special case of the RICI method for $R_c = 0$.

The RICI method was shown to be highly robust to suboptimal threshold values while outperforming the original ICI method in terms of the denoising accuracy [10], [11], [12], [13], [14], [15], [16], [17], as shown in the next section.

TABLE I

DENOISED RMSES FOR A RANGE OF NOISY SNRS WITH REGARDS TO SIGNAL SAMPLING PERIOD (100 MONTECARLO NOISE REALISATIONS, $\Gamma = 1.5$, $R_c = 0.82$).

SNR \ Δ	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$
	ICI					RICI					Improvement (RICI vs. ICI) [%]				
	Blocks														
5	1.482	0.999	0.770	0.381	0.271	0.906	0.653	0.598	0.540	0.534	**38.9**	34.6	22.4	-41.8	-97.2
10	1.431	0.920	0.713	0.350	0.250	0.620	0.318	0.231	0.114	0.089	56.7	65.5	**67.6**	67.5	64.4
15	1.413	0.884	0.692	0.339	0.241	0.566	0.286	0.201	0.095	0.068	59.9	67.6	70.9	**71.9**	71.8
20	1.406	0.870	0.681	0.335	0.239	0.551	0.267	0.189	0.089	0.064	60.8	69.4	72.3	**73.3**	73.0
30	1.408	0.863	0.676	0.333	0.237	0.541	0.255	0.183	0.087	0.062	61.6	70.4	72.9	**73.8**	73.6
	HeaviSine														
5	2.062	1.142	0.885	0.459	0.358	1.087	0.738	0.632	0.496	0.472	**47.3**	35.3	28.6	-8.0	-31.8
10	1.969	1.104	0.859	0.429	0.335	0.872	0.497	0.386	0.214	0.167	**55.7**	55.0	55.1	50.1	50.0
15	1.966	1.099	0.859	0.424	0.332	0.852	0.476	0.365	0.188	0.138	56.7	56.7	57.5	55.7	**58.5**
20	1.960	1.098	0.859	0.425	0.333	0.862	0.474	0.363	0.185	0.134	56.0	56.8	57.7	56.4	**59.7**
30	1.962	1.099	0.862	0.428	0.334	0.859	0.481	0.363	0.188	0.135	56.2	56.2	57.8	56.1	**59.7**
	Doppler														
5	0.244	0.166	0.134	0.078	0.060	0.137	0.094	0.081	0.057	0.051	**43.8**	43.5	39.4	26.4	15.8
10	0.240	0.163	0.130	0.074	0.057	0.111	0.074	0.060	0.034	0.026	53.9	**54.6**	54.3	53.7	53.8
15	0.238	0.162	0.130	0.073	0.057	0.106	0.071	0.057	0.031	0.024	55.3	56.3	56.4	57.4	**58.0**
20	0.238	0.162	0.130	0.073	0.057	0.104	0.070	0.056	0.031	0.023	56.2	56.7	56.5	57.8	**58.6**
30	0.238	0.162	0.130	0.074	0.057	0.103	0.070	0.057	0.031	0.024	56.6	57.0	56.4	57.6	**58.5**
	Bumps														
5	1.109	0.472	0.376	0.210	0.155	0.538	0.258	0.225	0.157	0.140	**51.5**	45.2	40.1	25.3	9.5
10	1.067	0.449	0.354	0.197	0.145	0.311	0.200	0.166	0.091	0.069	**70.9**	55.4	53.2	53.6	52.6
15	1.048	0.444	0.346	0.194	0.143	0.238	0.195	0.159	0.085	0.062	**77.3**	56.1	54.0	56.4	56.7
20	1.039	0.443	0.344	0.193	0.143	0.213	0.195	0.157	0.084	0.061	**79.5**	56.1	54.2	56.5	57.1
30	1.027	0.444	0.342	0.194	0.143	0.210	0.195	0.157	0.085	0.062	**79.6**	56.1	54.0	56.4	56.8

TABLE II

DENOISED MAE FOR A RANGE OF NOISY SNRS WITH REGARDS TO SIGNAL SAMPLING PERIOD (100 MONTECARLO NOISE REALISATIONS, $\Gamma = 1.5$, $R_c = 0.82$).

SNR \ Δ	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$	$\frac{1}{100}$	$\frac{1}{500}$	$\frac{1}{1000}$	$\frac{1}{5000}$	$\frac{1}{10000}$
	ICI					RICI					Improvement (RICI vs. ICI) [%]				
	Blocks														
5	1.301	0.815	0.604	0.291	0.206	0.637	0.397	0.346	0.289	0.280	51.0	**51.3**	42.7	0.7	-35.7
10	1.266	0.755	0.563	0.274	0.195	0.407	0.179	0.121	0.054	0.040	67.9	76.3	78.6	**80.3**	79.4
15	1.253	0.731	0.547	0.267	0.190	0.377	0.164	0.107	0.047	0.033	69.9	77.6	80.5	82.4	**82.6**
20	1.244	0.721	0.540	0.264	0.188	0.369	0.157	0.104	0.047	0.033	70.3	78.2	80.8	**82.4**	82.3
30	1.243	0.716	0.536	0.263	0.187	0.370	0.151	0.102	0.046	0.033	70.2	78.8	80.9	**82.4**	82.3
	HeaviSine														
5	1.817	0.992	0.754	0.383	0.290	0.871	0.553	0.455	0.313	0.276	**52.1**	44.3	39.7	18.1	4.9
10	1.740	0.948	0.723	0.349	0.261	0.733	0.412	0.316	0.170	0.133	**57.9**	56.5	56.3	51.1	49.3
15	1.738	0.942	0.721	0.341	0.251	0.723	0.398	0.297	0.144	0.104	58.4	57.8	**58.8**	57.8	58.6
20	1.732	0.940	0.719	0.340	0.251	0.728	0.397	0.295	0.137	0.097	58.0	57.7	59.0	59.6	**61.5**
30	1.735	0.941	0.721	0.342	0.250	0.727	0.402	0.294	0.136	0.093	58.1	57.3	59.2	60.3	**62.8**
	Doppler														
5	0.205	0.139	0.109	0.058	0.044	0.107	0.071	0.060	0.039	0.033	47.7	**49.2**	45.0	31.7	23.5
10	0.202	0.135	0.105	0.053	0.039	0.089	0.057	0.045	0.025	0.019	55.7	**58.1**	57.5	52.9	51.3
15	0.200	0.135	0.104	0.052	0.038	0.088	0.054	0.041	0.021	0.016	56.2	**60.1**	**60.1**	58.6	57.9
20	0.200	0.135	0.104	0.051	0.037	0.086	0.053	0.041	0.020	0.015	57.0	60.6	**60.7**	60.2	60.1
30	0.200	0.135	0.104	0.051	0.037	0.085	0.053	0.041	0.020	0.014	57.4	60.9	61.0	60.9	**61.2**
	Bumps														
5	0.751	0.335	0.269	0.142	0.104	0.350	0.154	0.131	0.089	0.078	53.4	**53.9**	51.4	37.3	24.9
10	0.739	0.331	0.260	0.138	0.100	0.199	0.117	0.095	0.052	0.039	**73.0**	64.5	63.7	62.4	61.1
15	0.728	0.332	0.257	0.137	0.100	0.146	0.118	0.095	0.051	0.038	**80.0**	64.5	63.0	62.6	62.2
20	0.726	0.332	0.257	0.138	0.100	0.119	0.119	0.097	0.053	0.039	**83.6**	64.0	62.3	61.7	61.5
30	0.722	0.332	0.257	0.138	0.101	0.100	0.120	0.098	0.054	0.040	**86.1**	63.7	61.8	61.0	60.6

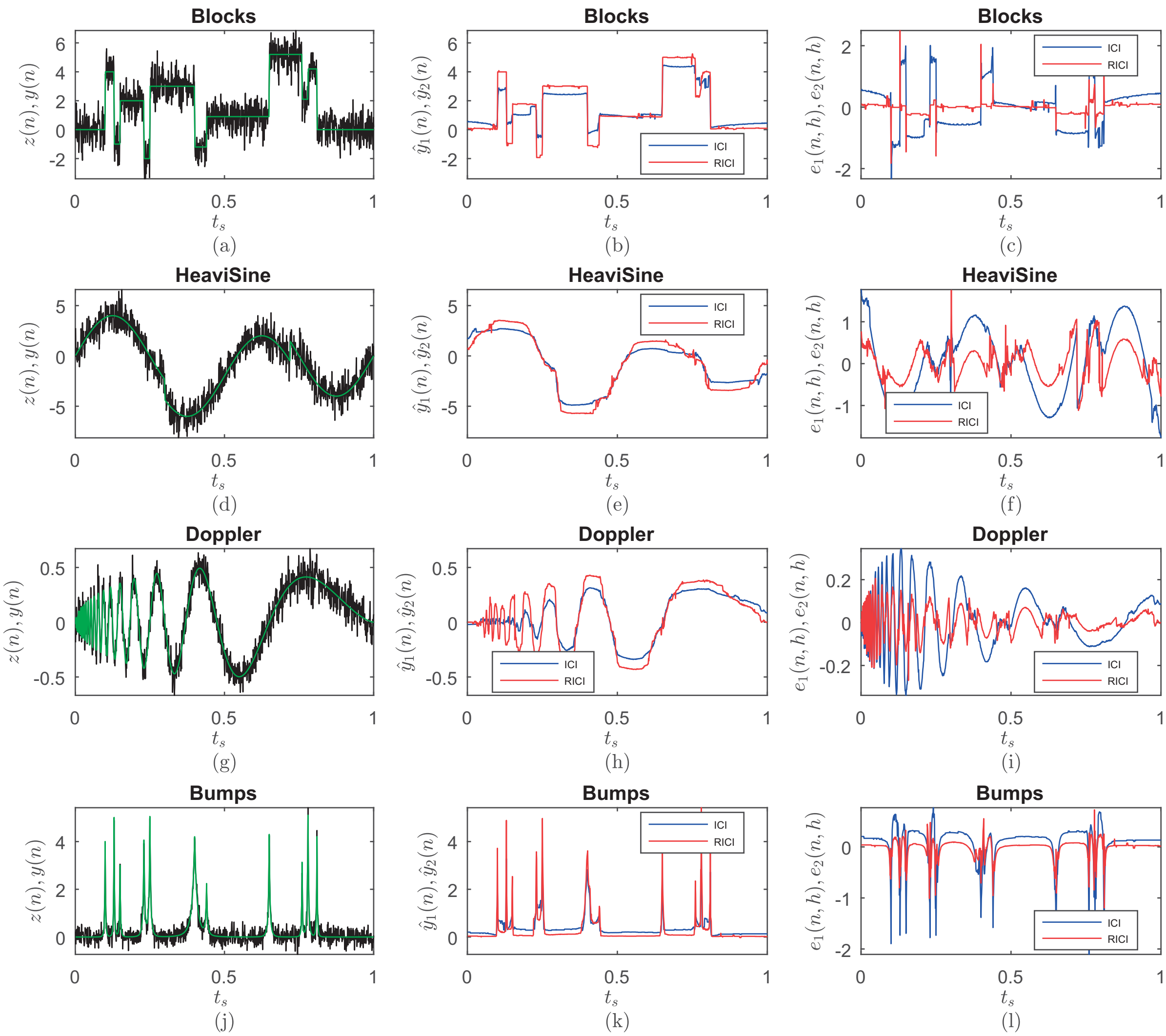

Fig. 1. Denoising results for tested signals (N=1000, $\Delta = \frac{1}{1000}$, $\Gamma = 1.5$, $R_c = 0.82$, noisy signal SNR=10). a) Noise-free $y(t_s)$ and noisy $z(t_s)$ Blocks signal. b) Blocks signal denoised using the ICI (blue, $\hat{y}_1(t_s)$) and RICI method (red, $\hat{y}_2(t_s)$). c) Estimation error for the Blocks signal for the ICI (blue, $e_1(t_s)$, RMSE=0.6954) and RICI method (red, $e_2(t_s)$, RMSE=0.2167). d) Noise-free $y(t_s)$ and noisy $z(t_s)$ HeaviSine signal. e) HeaviSine signal denoised using the ICI (blue, $\hat{y}_1(t_s)$) and RICI method (red, $\hat{y}_2(t_s)$). f) Estimation error for the HeaviSine signal for the ICI (blue, $e_1(t_s)$, RMSE=0.8451) and RICI method (red, $e_2(t_s)$, RMSE=0.3862). g) Noise-free $y(t_s)$ and noisy $z(t_s)$ Doppler signal. h) Doppler signal denoised using the ICI (blue, $\hat{y}_1(t_s)$) and RICI method (red, $\hat{y}_2(t_s)$). i) Estimation error for the Doppler signal for the ICI (blue, $e_1(t_s)$, RMSE=0.1329) and RICI method (red, $e_2(t_s)$, RMSE=0.0634). j) Noise-free $y(t_s)$ and noisy $z(t_s)$ Bumps signal. k) Bumps signal denoised using the ICI (blue, $\hat{y}_1(t_s)$) and RICI method (red, $\hat{y}_2(t_s)$). l) Estimation error for the Bumps signal for the ICI (blue, $e_1(t_s)$, RMSE=0.3500) and RICI method (red, $e_2(t_s)$, RMSE=0.1565).

III. Simulation Results

Denoising performances of the ICI and the improved RICI method with regards to signal sampling rate have been analyzed on four standard test signals (Blocks, HeaviSine, Doppler, and Bumps signal), each 1 s long. Denoising simulations (performed in Matlab 2016b) have been performed for 100 signal lengths of each tested signal $N = \{100, 200, 300, \cdots, 10000\}$ sampled with sampling periods $\Delta = \{\frac{1}{100}, \frac{1}{200}, \frac{1}{300}, \cdots, \frac{1}{10000}\}$, respectively. Estimation quality has been measured in terms of the reduction in denoised RMSE and MAE, averaged over 100 random MonteCarlo realizations of noise. The parameter Γ was set to 1.5 (belonging to an interval proposed in [3]) and optimal threshold $R_c = 0.82$ was calculated as the one minimizing RMSE.

Fig. 1 gives an example of denoising of noisy (SNR=10) signals Blocks, HeaviSine, Doppler, and Bumps for $N = 1000$ with $\Gamma = 1.5$ and optimal $R_c = 0.82$. Fig. 1(a) gives noisy and noise-free Blocks signals, followed by the signals denoised using the ICI and the RICI method shown in Fig. 1(b). Fig. 1(c) shows estimation error for the ICI and RICI method which was reduced, in terms of the RMSE, by 68.84% (from 0.6954 for the ICI to 0.2167 for the RICI method). Next, Fig. 1(d) shows noisy and noise-free HeaviSine signals. Fig. 1(e) show denoised HeaviSine signals obtained using the ICI and RICI method, followed by their estimation errors presented in Fig. 1(f). As it can be seen, estimation error RMSE was reduced from 0.8451 to 0.3862 (by 54.30%). Figs. 1(g)-1(i) show an

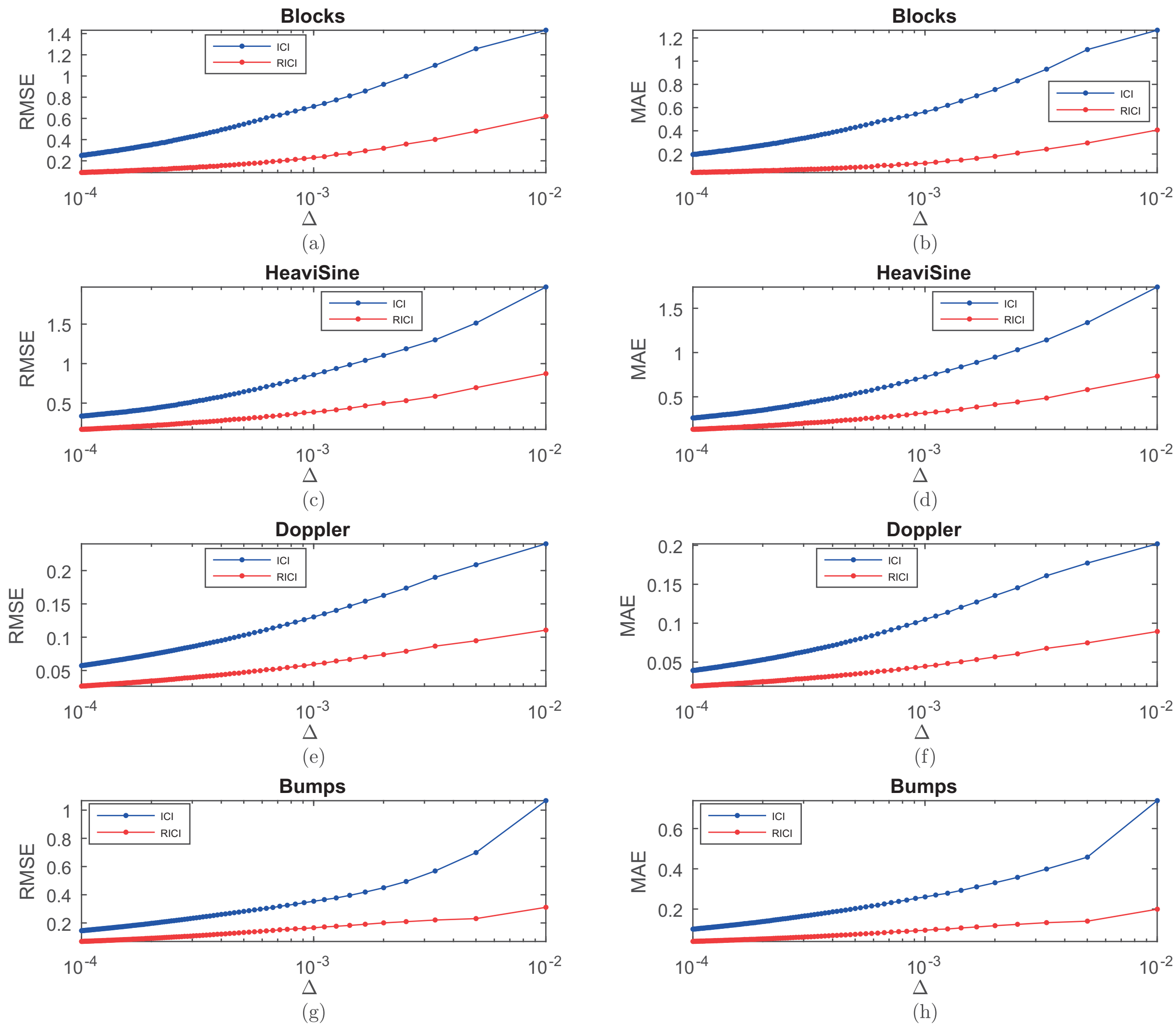

Fig. 2. Denoised signal RMSE and MAE for a range of signal sampling rates for the ICI (blue) and the RICI (red) method ($\Gamma = 1.5$, $R_c = 0.82$, noisy signal SNR=10, averaged over 100 MonteCarlo noise realisations). a) Denoised Blocks signal RMSE. b) Denoised Blocks signal MAE. c) Denoised HeaviSine signal RMSE. d) Denoised HeaviSine signal MAE. e) Denoised Doppler signal RMSE. f) Denoised Doppler signal MAE. g) Denoised Bumps signal RMSE. h) Denoised Bumps signal MAE.

example of denoising of Doppler signal. Namely, Figs. 1(g) and 1(h) show noisy and noise-free, as well as signals denoised by the ICI and RICI method, respectively. The estimation error for the ICI and RICI method for Doppler signal (which was reduced 52.32%, i.e. from 0.1329 for the ICI method to 0.0634 for the RICI method) is given in 1(i). Finally, noise-free and noisy Bumps signals are shown in Fig. 1(j). Denoised Bumps signals using the ICI and RICI method are shown in Fig. 1(k), followed by the estimation errors for both methods given in Fig. 1(l). As it can be seen, the RICI method has signif cantly outperformed the original ICI method in terms of the denoising quality for Bumps signal by reducing its estimation error by 55.30% (from 0.3500 to 0.1565).

Fig. 2 gives the RMSE and MAE for all tested signals with regards to sampling period (noisy signal SNR=10). Here is important to note that in all cases both data-driven adaptive methods exhibit signif cant increase in estimation quality for higher sampling rates (smaller sampling periods Δ). Namely, for the Blocks signal the RICI method outperformed the original ICI method by reducing estimation RMSE up to 56.70% and MAE up to 67.87%. For the HeaviSine signal, the RICI method reduced RMSE by up to 49.42% and MAE by up to 49.26%, when compared to the ICI method. Similar results were obtained for the Doppler signal when RMSE was reduced by up to 53.42% and MAE by up to 51.21%. Finally, for the Bumps signal the RMSE has been reduced by up to 52.44% and MAE by up to 60.80%.

Table I gives denoised RMSE for all tested signals for a range of noisy SNRs $\{5, 10, 15, 20, 30\}$ and sampling periods $\Delta = \left\{\frac{1}{100}, \frac{1}{500}, \frac{1}{1000}, \frac{1}{5000}, \cdots, \frac{1}{10000}\right\}$. Simulation results given in the Table I have been averaged over 100 MonteCarlo noise realisations. As presented in the table, the higher sampling rates (smaller sampling periods) have resulted in reduction of estimation RSME for all testes SNRs and all tested signals (both for the ICI and the RICI method). Furthermore, the RICI algorithm outperformed the original ICI algorithm in terms of

RMSE reduction in almost all cases. Namely, the RICI method has reduced the RMSE by up to 73.8% for the Blocks signal, by up to 59.7% for the HeaviSine signal, by up to 58.6% for the Doppler signal, and by up to 79.6% for the Bumps signal. In addition, denoising results for all tested signals in terms of the MAE are presented in Table II for a range of noisy SNRs and signal sampling periods (averaged over 100 MonteCarlo noise simulations, $\Gamma = 1.5$, $R_c = 0.82$). As it can be seen, for all tested SNRs both the ICI and RICI method ($\Gamma = 1.5$, $R_c = 0.82$) resulted in smaller MAE with increase in sampling rate (for smaller sampling periods). In other words, both the ICI and RICI method perform better for longer signals (signals sampled with higher frequency) in terms of denoising quality. In particular, the RICI method outperformed the ICI method in almost all cases reducing the MAE by up to 82.6% for the Blocks signal, by up to 62.8% for the HeaviSine signal, by up to 61.2% for the Doppler signal, and by up to 86.1% for the Bumps signal.

In conclusion, the given study elaborates on the performances of the adaptive data-driven ICI and the RICI methods with regards to sampling rate. The analysis has been been performed for several test signals and a range of SNRs. As shown in the paper, both methods (especially the improved RICI method) significantly improve estimation quality with increase of the signal resolution (for larger amount of data processed).

IV. Conclusion

This paper provides detailed elaboration on the denoising accuracy of the ICI-based and the RICI based adaptive temporal filtering methods with regards to signal sampling rate. The analysis was performed on four signal classes for a range of SNRs. As shown in the paper, the improved RICI method outperformed the original ICI method reducing the RSME by up 79.6% and MAE by up to 86.1%. Furthermore, both methods, especially the RICI method, were shown to exhibit estimation accuracy improvement with increase in signal sampling rate.

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